TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 1 2019

Section 1 Calculator Free

Combinatorics and Vector Basics

STUDENT'S NAME SOLUTIONS

DATE: Wednesday 6 March **TIME:** 15 minutes **MARKS**: 15

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (2 marks)

Express the following in factorial form

$$\frac{14 \times 13}{3 \times 2 \times 1} = \frac{14!}{12!\,3!}$$

2. (5 marks)

(a) Prove that ${}^{n}C_{r} = \frac{n}{r} \times {}^{n-1}C_{r-1}$ [3]

$$LHS = \frac{n!}{r!\,(n-r)!}$$

$$= \frac{n \cdot (n-1)!}{r \cdot (r-1)! \times (n-1-r+1)!}$$

$$= \frac{n}{r} \times \frac{(n-1)!}{(r-1)!\,(n-1-(r-1))!}$$

$$= \frac{n}{r} \times {}^{n-1}C_{r-1} = RHS \quad QED$$

(b) Given that ${}^{14}C_5 = 2002$ and ${}^{15}C_5 = 3003$, determine ${}^{15}C_6$ [2]

$${}^{15}C_6 = \frac{15}{6} \times {}^{14}C_5$$

$$= \frac{5}{2} \times 2002$$

$$= 5005$$

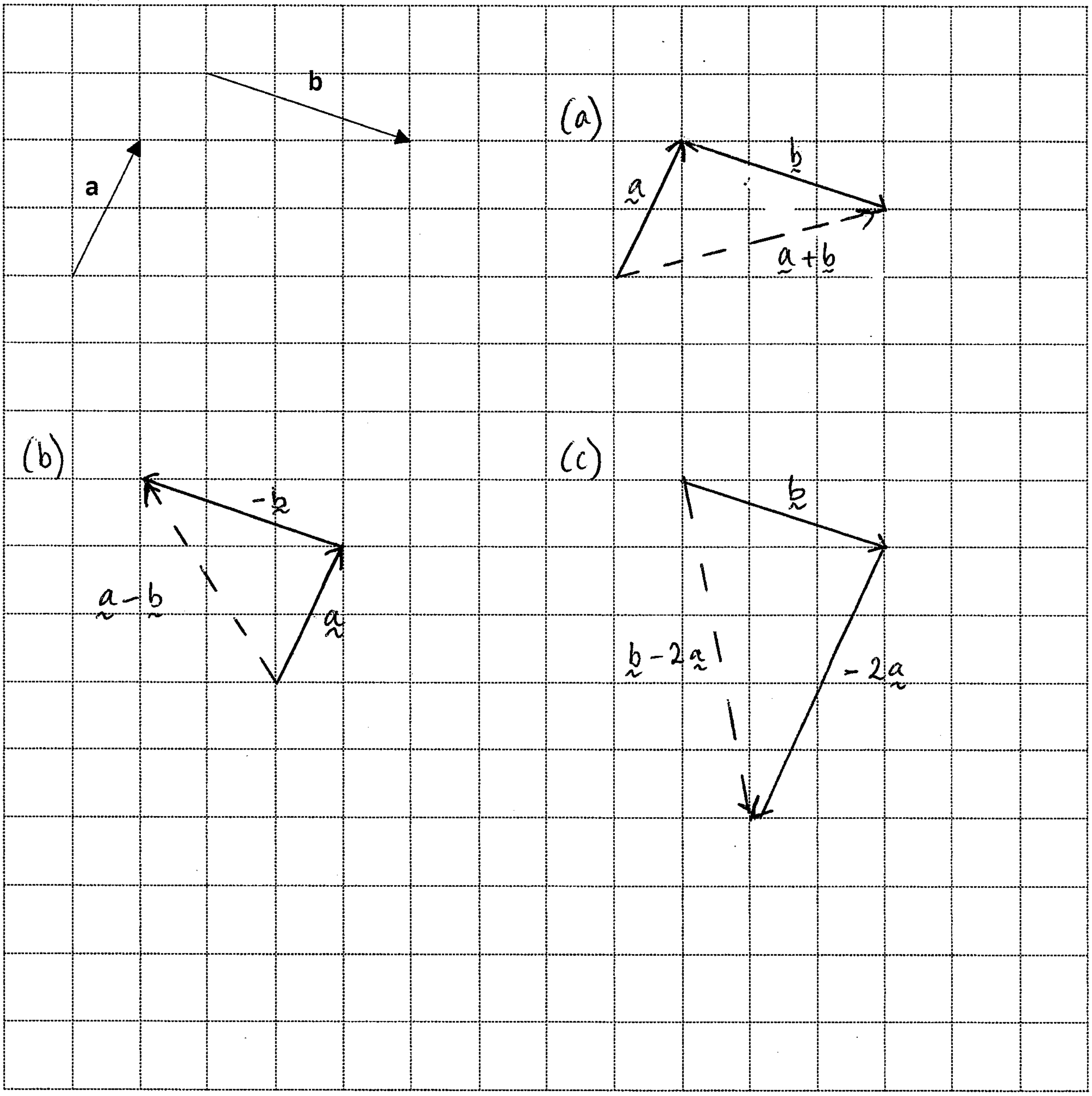
3. (4 marks)

On the grid below draw the following, given **a** and **b** as shown.

(a) **a** + **b** [1]

(b) **a** – **b** [1]

(c) **b** – 2**a** [2]



4. (4 marks)

C, Y, L, I, A

The letters of the word CYCLICAL are rearranged in a line. Determine the total number of 3 letter "words" that can be formed.

All different $\quad {}^5C_3 \times 3! = 60$

2 L's $\quad {}^4C_1 \times \frac{3!}{2!} = 12$

2 C's $\quad {}^4C_1 \times \frac{3!}{2!} = 12$

3 C's $\quad = 1$

$= 85$ "words"

TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 1 2019

Section 2 Calculator Assumed
Combinatorics and Vector Basics

STUDENT'S NAME SOLUTIONS

DATE: Wednesday 6 March **TIME:** 30 minutes **MARKS**: 35

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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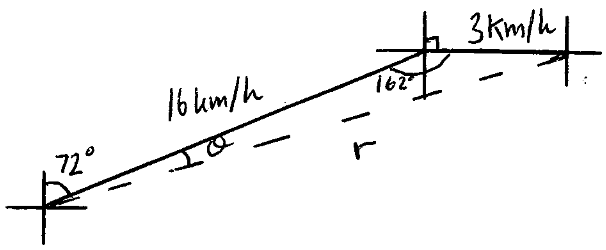
5. (4 marks)

An ocean liner is travelling at 16 km/h on a course of 072°. However, it is drifting off-course due to a 3 km/h ocean current which is flowing from the west. What is the actual speed and direction of the ocean liner?

Draw a diagram to assist you.



$|\underset{\sim}{r}|^2 = 16^2 + 3^2 - 2 \times 16 \times 3 \times \cos(162)$

$|\underset{\sim}{r}| = 18.88 \text{ km/h}$

$\dfrac{3}{\sin\theta} = \dfrac{18.88}{\sin(162)}$

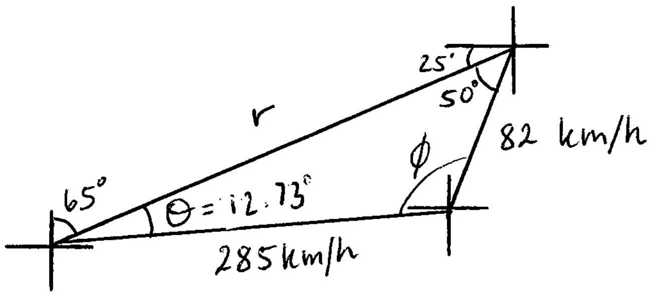
$\theta = 2.81°$

$\therefore$ 18.88 km/h @ 74.81°

6. (7 marks)

In still air an aircraft can maintain a speed of 285 km/h. The pilot wishes to fly the aircraft from Sydney to Fiji which is 1260 km away on a bearing of 065°. There is a wind blowing at 82 km/h from 195°.

(a) Draw a diagram to show all this information. The diagram does not have to be to scale. [2]



(b) Determine the bearing on which the pilot should steer the aircraft so that is flies directly to its destination. [3]

$$\frac{82}{\sin\theta} = \frac{285}{\sin(50)}$$

$$\theta = 12.73°$$

$$\text{Bearing} = 65 + 12.73$$
$$= 077.73°T$$

(c) How long will the journey take, to the nearest minute? [2]

$$\frac{|r|}{\sin(117.27)} = \frac{285}{\sin(50)} \qquad \phi = 117.27$$

$$|r| = 330.7 \text{ km/h}$$

$$t = \frac{1260}{330.7}$$

$$t = 3.81$$
$$= 3 \text{ hrs, } 49 \text{ mins}$$

7. (6 marks)

A passcode with 5-digits are made using the digits from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. No repeats in digits are allowed.

(a) How many different passcodes are possible? [1]

$^{10}P_5 = 30240$

(b) How many of the passcodes start with the digit 4 or end with the digit 9? [2]

$= \underline{1} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} + \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{1} - \underline{1} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{1}$

$= 5712$

(c) How many of the passcodes are even and greater than 60000? [3]

Start $\bar{w}$ even

$\underline{2} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{4}$

$+$

Start $\bar{w}$ odd

$\underline{2} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5}$

$= 6048$

8. (6 marks)

A committee of 9 people are to be selected from 10 Labor, 8 Liberal and 5 Greens politicians. How many different ways can the committee be selected if:

(a) there are no restrictions? [1]

$^{23}C_9 = 817190$

(b) the liberal representatives are in the majority? [3]

$= \binom{8}{5}\binom{15}{4} + \binom{8}{6}\binom{15}{3} + \binom{8}{7}\binom{15}{2} + \binom{8}{8}\binom{15}{1}$

$= 90035$

(c) a husband and wife pair, Alex and Alice, cannot be in the same committee? [2]

$= \binom{23}{9} - \binom{2}{2}\binom{21}{7}$

$= 700\,910$

9. (4 marks)

Three Physics books, four Chemistry books and two Mathematics books are to be arranged in a book shelf. Determine the number of arrangements with either a Physics book on the extreme left or a Chemistry book exactly in the middle or a Mathematics book on the extreme right.

P : Physics on left

C : Chemistry in middle

M : Maths on right

$n(P) = 3 \times 8! = 120960$

$n(C) = 4 \times 8! = 161280$

$n(M) = 2 \times 8! = 80640$

$n(P \cap C) = 3 \times 4 \times 7! = 60480$

$n(P \cap M) = 3 \times 2 \times 7! = 30240$

$n(C \cap M) = 4 \times 2 \times 7! = 40320$

$n(P \cap C \cap M) = 3 \times 4 \times 2 \times 6! = 17280$

$\therefore n(P \cup C \cup M) = 120960 + 161280 + 80640 - 60480 - 30240 - 40320 + 17280$

$= 249120$

10. (8 marks)

Hamish and Andy's Bitcoin account is protected by a 4 character password. The characters are chosen from the 26 letters of the alphabet (not case sensitive) and the digits 0 to 9 inclusive.

How many different passwords are there if

(a) letters of the alphabet and digits are used [1]

$$36^4 = 1679616$$

(b) two letters and two digits are used, no character being used more than once [2]

$$= \binom{26}{2}\binom{10}{2} \times 4! = 351000$$

(c) more letters than digits are used, no character being used more than once [3]

$$= \binom{26}{3}\binom{10}{1} \times 4! + \binom{26}{4}\binom{10}{0} \times 4!$$

$$= 982800$$

(d) there must be exactly two letters and the letters must be consecutive and adjacent and in ascending order, no character being used more than once [2]

$$= \binom{25}{1}\binom{10}{2} \times 3!$$

$$= 6750$$